Two-sided bounds on minimum-error quantum measurement, on the reversibility of quantum dynamics, and on the quantum conditional min-entropy using directional iterates

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In a unified framework we estimate the following quantities of interest in quantum information theory:

1. The minimum-error quantum distinguishability of arbitrary ensembles of mixed quantum states.

2. The approximate reversibility of quantum dynamics in terms of entanglement fidelity. (This is referred to as "channel-adapted quantum error recovery" when applied to the composition of an encoding operation and a noise channel.)

3. The maximum overlap between a bipartite pure quantum state and a bipartite mixed state that may be achieved by applying a local quantum operation to one part of the mixed state.

4. The conditional min-entropy of bipartite quantum states.

Our primary tools are simple geometric arguments and “small angle” initialization of an abstract generalization of the iterative schemes of Ježek-Řeháček-Fiurášek [Phys. Rev. A **65,** 060301], Ježek-Fiurášek-Hradil [Phys. Rev. A **68**, 012305], and Reimpell-Werner [Phys. Rev. Lett. **94**, 080501]. This work may be seen as a generalization of Holevo’s asymptotically-optimal measurement for distinguishing pure states in 1978 and of the related distinguishability bounds proved by Paul Curlander in 1979. (Holevo’s quadratically-weighted pure-state measurement has often been confused with the linearly-weighted “pretty good” measurement of Belavkin (1975) and Hausladen-Wootters (1994), which is NOT asymptotically optimal in the sense of Holevo.) As corollaries, a conjecture of Schumacher and Westmoreland is resolved and the monotonicity of Reimpell and Werner’s “channel-power” iteration follows in greater generality.